Calculus Formula Sheet

Derivatives · Integrals · Limits · Series · Theorems · Applications

Derivatives

Power rule	$\frac{d}{dx}x^n = nx^{n-1}$
Product rule	$(fg)' = f'g + fg'$
Quotient rule	$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$
Chain rule	$\frac{d}{dx}f(g(x)) = f'(g(x))\,g'(x)$
Sine & cosine	$(\sin x)' = \cos x$ $(\cos x)' = -\sin x$
Tangent & secant	$(\tan x)' = \sec^2 x$ $(\sec x)' = \sec x \tan x$
Exponentials	$(e^x)' = e^x$ $(a^x)' = a^x \ln a$
Natural log	$(\ln x)' = \frac{1}{x}$
Inverse trig	$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$ $(\arctan x)' = \frac{1}{1+x^2}$

Integrals

Power rule	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ $n \neq -1$
Reciprocal	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$
Exponentials	$\int e^x\,dx = e^x + C$ $\int a^x\,dx = \frac{a^x}{\ln a} + C$
Sine & cosine	$\int \sin x\,dx = -\cos x + C$ $\int \cos x\,dx = \sin x + C$
Secant squared	$\int \sec^2 x\,dx = \tan x + C$
Arctangent form	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
Arcsine form	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
Integration by parts	$\int u\,dv = uv - \int v\,du$
u-substitution	$\int f(g(x))\,g'(x)\,dx = \int f(u)\,du$

Limits

Sine limit	$\lim_{x\to 0}\frac{\sin x}{x} = 1$
Cosine limit	$\lim_{x\to 0}\frac{1-\cos x}{x} = 0$
Exponential limit	$\lim_{x\to 0}\frac{e^x - 1}{x} = 1$
Definition of e	$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$
L'Hôpital's rule	$\lim\frac{f(x)}{g(x)} = \lim\frac{f'(x)}{g'(x)}$ for $\frac{0}{0}, \frac{\infty}{\infty}$

Derivative definition	$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

Series

Geometric series	$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$ $\lvert r\rvert < 1$
p-series	$\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges $\iff p > 1$
Ratio test	$L = \lim_{n\to\infty}\left\lvert\frac{a_{n+1}}{a_n}\right\rvert$ $L < 1$ converges, $L > 1$ diverges
Taylor series	$f(x) = \sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$
Exponential	$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}$
Sine	$\sin x = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}$
Cosine	$\cos x = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$
Natural log	$\ln(1+x) = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^n}{n}$ $-1 < x \leq 1$

Theorems

Fundamental Theorem I	$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$
Fundamental Theorem II	$\int_a^b f(x)\,dx = F(b) - F(a)$
Mean Value Theorem	$f'(c) = \frac{f(b)-f(a)}{b-a},\ c \in (a,b)$
Rolle's Theorem	$f(a) = f(b)$ $\exists\, c \in (a,b) : f'(c) = 0$
Intermediate Value Thm.	$f(a) < N < f(b)$ $\exists\, c \in (a,b) : f(c) = N$

Applications

Arc length	$L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$
Disk method	$V = \pi\int_a^b [f(x)]^2\,dx$
Washer method	$V = \pi\int_a^b \left(R(x)^2 - r(x)^2\right)dx$
Shell method	$V = 2\pi\int_a^b x\,f(x)\,dx$
Average value	$f_{\text{avg}} = \frac{1}{b-a}\int_a^b f(x)\,dx$
Newton's method	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
Linear approximation	$L(x) = f(a) + f'(a)(x-a)$